

# Unconventional behavior of Dirac fermions in three-dimensional gauge theory

Jing Wang<sup>1</sup> and Guo-Zhu Liu<sup>2,1</sup>

<sup>1</sup>*Department of Modern Physics, University of Science and Technology of China, Hefei, Anhui, 230026, P.R. China*

<sup>2</sup>*Max Planck Institut für Physik komplexer Systeme, D-01187 Dresden, Germany*

We study unconventional behavior of massless Dirac fermions due to interaction with a U(1) gauge field in three time-space dimensions. At zero chemical potential, the longitudinal and transverse components of gauge interaction are both long-ranged. There is no fermion velocity renormalization since the system respects Lorentz invariance. At finite chemical potential, the Lorentz invariance is explicitly broken by the finite Fermi surface. The longitudinal gauge interaction is statically screened and becomes unimportant, whereas the transverse gauge interaction remains long-ranged and leads to singular renormalization of fermion velocity. The anomalous dimension of fermion velocity is calculated by means of renormalization group method. We then examine the influence of singular velocity renormalization on several physical quantities, and show that they exhibit different behavior at zero and finite chemical potential.

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## I. INTRODUCTION

Quantum electrodynamics of massless Dirac fermions defined in three time-space dimensions (QED<sub>3</sub>) has been extensively investigated for thirty years [1–23]. Different from its four-dimensional counterpart, QED<sub>3</sub> is a super-renormalizable field theory, so its ultraviolet behavior can be well controlled. This gauge field theory is known to exhibit asymptotic freedom [10], which means the gauge interaction becomes stronger at lower energies. Due to this feature, intriguing non-perturbative phenomena are expected to occur in the low-energy regime. When Dirac fermions are strictly massless, the Lagrangian respects a continuous chiral symmetry. However, the strong gauge interaction may trigger fermion-antifermion vacuum condensation,  $\langle\bar{\psi}\psi\rangle \neq 0$ , which generates a finite fermion mass and induces dynamical chiral symmetry breaking (DCSB). Appelquist *et al.* first found that DCSB can occur when the fermion flavor  $N$  is smaller than some critical value  $N_c = 32/\pi^2$  [3]. Motivated by this very interesting prediction, intense theoretical effort has been devoted to studying this problem [4–7, 9–20]. Despite some debate [1, 6, 7], most of these analytical and numerical calculations agree that a critical value exists at roughly  $N_c \approx 3.5$ . A remarkable consequence of DCSB is that it leads to weak confinement [8, 9].

Apart from exhibiting many properties interesting in the context of elementary particle physics, QED<sub>3</sub> also has extensive applications in condensed matter physics. Specifically, it serves as an effective low-energy theory of  $d$ -wave high-temperature superconductor [14, 24–38] and some quantum spin liquid state [39]. The happening of DCSB in QED<sub>3</sub> corresponds to the formation of two-dimensional long-range antiferromagnetic order [14, 28, 32, 33, 35]. Recently, it is proposed that QED<sub>3</sub> may be simulated on optical lattice [40], which provides an opportunity of measuring DCSB experimentally.

When the fermion flavor is beyond the threshold,  $N > N_c$ , no dynamical fermion mass can be generated. There is no fermion vacuum condensation, i.e.,  $\langle\bar{\psi}\psi\rangle = 0$ , and

the continuous chiral symmetry is preserved. However, the absence of chiral condensation in the vacuum does not mean that the chiral symmetric phase is trivial. On the contrary, many highly nontrivial features can emerge in the symmetric phase of QED<sub>3</sub>. In particular, the gauge interaction is able to cause breakdown of Fermi liquid (FL). In 1973, Holstein *et al.* showed [41] that the unscreened transverse component of electromagnetic field in (3+1)-dimensional non-relativistic electron gas leads to unusual logarithmic specific heat,  $\propto T \ln T$ , which is apparently out of the scope of FL theory. This discovery have stimulated extensive investigations of non-FL behavior in various gauge theories, in the contexts of both condensed matter physics [27, 42–53] and particle physics [54–58]. The non-FL behavior of massless Dirac fermions in QED<sub>3</sub> has also been discussed [21, 22, 31]. In addition, QED<sub>3</sub> can be used to describe some intriguing states, such as algebraic spin liquid state [29–33, 36] and algebraic charge liquid state [37].

In some many-particle systems described by QED<sub>3</sub>, there is a finite density of massless Dirac fermions. The finite fermion density is usually represented by chemical potential  $\mu$ . An interesting question is how this chemical potential affects the physical properties of QED<sub>3</sub>. The impacts of finite chemical potential on DCSB have been addressed in [17, 18, 20], where it is found that the critical flavor  $N_c$  is lowered as chemical potential grows and DCSB is completely suppressed when chemical potential is sufficiently large. In the chiral symmetric phase with strictly massless Dirac fermions, the gauge interaction also leads to different properties at zero and finite chemical potential. For instance, the Dirac fermion damping rate behaves as  $\propto \omega^{1/2}$  at zero chemical potential [21] and  $\propto \omega^{2/3}$  at finite chemical potential [22].

In this paper, we consider chiral symmetric phase of QED<sub>3</sub> and study unconventional behavior of massless Dirac fermions at finite chemical potential. We assume a relatively large fermion flavor  $N$  so that the fermions are strictly massless. One important effect of a finite chemical potential is that it explicitly breaks the Lorentz in-

variance. As a consequence, the longitudinal component of gauge field develops an effective mass that is proportional to the chemical potential  $\mu$ , which is analogous to the static screening of Coulomb interaction. However, the transverse component of gauge field remains massless due to the gauge invariance. Because of such difference in the longitudinal and transverse components of gauge field, the temporal and spatial components of fermion self-energy are no longer identical, which in turn gives rise to nontrivial renormalization of fermion velocity.

We shall analyze fermion velocity renormalization by performing a renormalization group (RG) calculation [59–62]. The fermion velocity remains a constant at zero chemical potential after including the gauge interaction corrections since the Lorentz invariance is preserved. However, it acquires strong momentum dependence after developing an anomalous dimension  $\gamma_v$  at finite chemical potential. The appearance of nonzero anomalous dimension is a consequence of Lorentz symmetry breaking and gauge symmetry. Therefore, QED<sub>3</sub> with a finite chemical potential is fundamentally different from that defined at zero chemical potential. We then evaluate specific heat, density of states (DOS), and compressibility of massless fermions using  $\gamma_v$  obtained in the RG analysis. After analytical and numerical calculations, we demonstrate that massless Dirac fermions exhibits unconventional, non-FL like, behavior at finite chemical potential.

The rest of this paper is organized as follows. We define the Lagrangian and then perform RG analysis of fermion velocity renormalization in Sec. II. A finite anomalous dimension of fermion velocity is obtained. We calculate specific heat in Sec. III, and DOS and compressibility in Sec. IV. In section V, we briefly summarize the results.

## II. RENORMALIZATION GROUP ANALYSIS OF FERMI VELOCITY RENORMALIZATION

The Lagrangian density for QED<sub>3</sub> with  $N$ -flavor Dirac fermions is given by

$$\mathcal{L} = \sum_{i=1}^N \bar{\psi}_i [(\partial_\tau - \mu - ie a_0) \gamma_0 - i v_F \boldsymbol{\gamma} \cdot (\partial - ie \mathbf{a})] \psi_i - \frac{1}{4} F_{\mu\nu}^2, \quad (1)$$

where  $\mu$  represents the chemical potential and  $v_F$  is the constant fermion velocity. The Dirac fermions can be described by a four-component spinor field  $\psi$  and  $\bar{\psi} = \psi^\dagger \gamma^0$ . The  $4 \times 4$   $\gamma$  matrices can be chosen as:  $\gamma_\mu = (\sigma_3, \sigma_1, \sigma_2) \otimes \sigma_3$ , which satisfy the standard Clifford algebra  $\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}$  with metric  $g_{\mu\nu} = \text{diag}(1, 1, 1, 1)$ . In this paper, we consider a large  $N$  and perform  $1/N$  expansion. For convenience, we work in units with  $\hbar = k_B = 1$  and restore them whenever necessary.

It is now helpful to further remark on the physical meaning of fermion velocity  $v_F$ . There are two ways to define the velocity  $v_F$ . If we regard QED<sub>3</sub> as a standard relativistic quantum field theory, the velocity of massless particles is simply the velocity of light, i.e.,  $v_F \equiv c$ . If, on

the other hand, we consider the effective QED<sub>3</sub> theory that is derived from a microscopic model of some condensed matter system (for instance, Hubbard model of high-temperature superconductor) [24–26], the fermion velocity should be calculated from the band structure of the corresponding microscopic model. In the latter case, the fermion velocity  $v_F$  is no longer equal to the velocity of light,  $c$ . Its magnitude is strongly material dependent, but is always much smaller than  $c$  [24–26]. In the present paper, we assume a constant velocity  $v_F$  and study its renormalization due to gauge interaction at finite chemical potential. The main conclusion depends only on the momentum dependence of the renormalized velocity, but not on the concrete magnitude of the constant  $v_F$ .

In Euclidian space, the free propagator of massless Dirac fermions at zero  $\mu$  is

$$G_0(k) = \frac{1}{\not{k}} = \frac{\gamma_0 k_0 + v_F \boldsymbol{\gamma} \cdot \mathbf{k}}{k^2}. \quad (2)$$

At finite  $\mu$ , it proves convenient to work in the Matsubara formalism and write the fermion propagator as

$$G_0(i\omega_n, \mathbf{k}) = \frac{1}{(i\omega_n + \mu)\gamma_0 - v_F \boldsymbol{\gamma} \cdot \mathbf{k}}, \quad (3)$$

where the fermion frequency is  $i\omega_n = i(2n+1)\pi/\beta = i(2n+1)\pi T$  with  $n$  being integers. For notational convenience, we use  $k_0$  to denote the imaginary frequency  $i\omega_n$ , so that the fermion propagator can also be written as

$$G_0(k) = \frac{1}{\not{k}} = \frac{\gamma_0(k_0 + i\mu) + v_F \boldsymbol{\gamma} \cdot \mathbf{k}}{k^2}. \quad (4)$$

### A. $\mu = 0$

We first consider the case of zero  $\mu$ . The key physical quantity is the fermion self-energy function, which can be used to calculate the RG flow of fermion velocity  $v_F$  [60–62]. Following the RG strategy presented in Ref. [60], we introduce two cutoffs  $\Lambda_0$  and  $\Lambda_1$ , with  $\Lambda_1$  being smaller than  $\Lambda_0$ . To the leading order of  $1/N$  expansion, the one-loop fermion self-energy at zero  $\mu$  is

$$\begin{aligned} \Sigma(k) &= \frac{\alpha}{N} \int_{\Lambda_1}^{\Lambda_0} \frac{d^3 q}{(2\pi)^3} \gamma_\mu \frac{(\not{k} - \not{q})}{(k-q)^2} \gamma_\nu D_{\mu\nu}(q) \\ &\equiv \gamma_0 k_0 \Sigma_0 + v_F \gamma_i k_i \Sigma_1. \end{aligned} \quad (5)$$

The full gauge field propagator  $D_{\mu\nu}(q)$  is given by the Dyson equation

$$D_{\mu\nu}^{-1}(q) = D_{\mu\nu}^{(0)-1}(q) + \Pi_{\mu\nu}(q), \quad (6)$$

with free gauge field propagator

$$D_{\mu\nu}^{(0)}(q) = \frac{1}{q^2} \left( g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right), \quad (7)$$

in the Landau gauge. To the leading order of  $1/N$  expansion, the one-loop contribution to vacuum polarization tensor  $\Pi_{\mu\nu}$  is

$$\begin{aligned}\Pi_{\mu\nu}(q) &= -\alpha \int \frac{d^3k}{(2\pi)^3} \frac{\text{Tr}[\gamma_\mu \not{k} \gamma_\nu (\not{q} + \not{k})]}{k^2(q+k)^2} \\ &= \Pi(q^2) \left( g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right).\end{aligned}\quad (8)$$

Here, it is convenient to define  $\alpha = Ne^2$ , which is fixed as  $N$  taken to be large [3]. Now the gauge field propagator becomes

$$D_{\mu\nu}(q) = \frac{1}{q^2 + \Pi(q)} \left( g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right), \quad (9)$$

where  $\Pi(q) = \frac{\alpha q}{8}$  at zero temperature.

According to the Dyson equation,  $G^{-1}(k) = G_0^{-1}(k) - \Sigma(k)$ , the full fermion propagator is

$$\begin{aligned}G(k) &= \frac{1}{\not{k} - \Sigma(k)} \\ &= \frac{1}{(1 - \Sigma_0) \gamma_0 k_0 + (1 - \Sigma_1) v_F \gamma_i k_i}.\end{aligned}\quad (10)$$

Apparently,  $(1 - \Sigma_0)$  correspond to the wave function renormalization, whereas  $(1 - \Sigma_1)$  represents the product of the renormalization factors of wave function and fermion velocity.

By inserting Eq. (9) into Eq. (5), it is easy to find that  $\Sigma_0 = \Sigma_1$ . Apparently, the temporal and spatial parts of fermion propagator are equally renormalized, which originates from the Lorentz invariance of the system at zero  $\mu$ . Therefore, the fermion velocity  $v_F$  is a flow-invariant constant.

### B. $\mu \neq 0$

At finite finite chemical potential, QED<sub>3</sub> theory can exhibit qualitatively new features. First of all, there appears a finite Fermi surface, which explicitly breaks the Lorentz invariance. In this case, the temporal and spatial components of fermion self-energy are no longer equivalent, so the fermion velocity may be singularly renormalized.

To proceed, we first need to discuss the effects of a finite chemical potential  $\mu$  on the effective gauge interaction function. These effects are reflected in the vacuum polarization function  $\Pi(q, \mu)$ . It is technically hard to obtain an entirely analytical expression for  $\Pi(q, \mu)$ , so we will derive an approximate  $\Pi(q, \mu)$  that captures the essential features of QED<sub>3</sub> at finite  $\mu$ .

Under the Coulomb gauge condition,  $q_i \Pi_{ij}(q) = 0$ , the vacuum polarization tensor can be decomposed into two independent parts [63]:

$$\Pi_{\mu\nu}(q_0, \mathbf{q}, \beta) = \Pi_A(q_0, \mathbf{q}, \beta) A_{\mu\nu} + \Pi_B(q_0, \mathbf{q}, \beta) B_{\mu\nu}, \quad (11)$$

where

$$A_{\mu\nu} = \delta_{\mu 0} \delta_{\nu 0}, \quad (12)$$

$$B_{\mu\nu} = \delta_{\mu i} \left( \delta_{ij} - \frac{q_i q_j}{\mathbf{q}^2} \right) \delta_{j\nu}, \quad \{i, j = 1, 2\}. \quad (13)$$

They are orthogonal and related by

$$A_{\mu\nu} + B_{\mu\nu} = \delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}. \quad (14)$$

Now, the full gauge field propagator  $D_{\mu\nu}(q_0, \mathbf{q}, \beta)$  can be written as

$$D_{\mu\nu}(q_0, \mathbf{q}, \beta) = \frac{A_{\mu\nu}}{\mathbf{q}^2 + \Pi_A(q_0, \mathbf{q}, \beta)} + \frac{B_{\mu\nu}}{q^2 + \Pi_B(q_0, \mathbf{q}, \beta)}, \quad (15)$$

where the functions  $\Pi_A$  and  $\Pi_B$  are related to the temporal and spatial components of vacuum polarization tensor  $\Pi_{\mu\nu}$  by

$$\Pi_A = \Pi_{00}, \quad (16)$$

$$\Pi_B = \Pi_{ii} - \frac{q_0^2}{\mathbf{q}^2} \Pi_{00}, \quad (17)$$

with  $q^2 = q_0^2 + \mathbf{q}^2$ . Based on these quantities, it is convenient to write the fermion self-energy as

$$\Sigma(k) \equiv \Sigma_A + \Sigma_B \equiv \gamma_0 k_0 \Sigma_0 + v_F \gamma_i k_i \Sigma_1, \quad (18)$$

where

$$\Sigma_0 = \Sigma_{A0} + \Sigma_{B0}, \quad (19)$$

$$\Sigma_1 = \Sigma_{A1} + \Sigma_{B1}. \quad (20)$$

In an earlier publication [22], the full analytical expressions of polarization function  $\Pi(q, \mu)$  was derived. These analytical expressions are too complicated and can not be directly used. It is necessary to make proper approximations. We first consider the temporal component  $\Pi_{00}(q, \mu)$ . In order to simplify computations, we utilize the so-called instantaneous approximation [63], i.e.,  $q_0 = 0$ , and write  $\Pi_{00}(q, \mu)$  approximately as

$$\Pi_{00}(\mathbf{q}, \mu) = \begin{cases} \frac{\alpha \mu}{8}, & \mu \geq \frac{|\mathbf{q}|}{2} \\ \frac{\alpha |\mathbf{q}|}{8}, & \mu < \frac{|\mathbf{q}|}{2} \end{cases}. \quad (21)$$

Apparently, the finite  $\mu$  serves as an energy scale:  $\Pi_{00}(q, \mu)$  behaves quite differently above and below  $2\mu$ . In the low-energy (long wavelength) limit,  $\mathbf{q} \rightarrow 0$ , we have

$$\Pi_{00}(\mathbf{q} \rightarrow 0, \mu) \neq 0. \quad (22)$$

As a consequence, the temporal part of gauge interaction becomes short-ranged due to static screening. In other words, the temporal component of gauge field acquires a finite effective mass that is proportional to  $\mu$ . Such short-ranged interaction does not lead to any singular contribution to fermion self-energy, and thus can be simply neglected.

We then consider the spatial component of polarization function  $\Pi_{ii}(q, \mu)$ . Similar to its temporal counterpart,  $\Pi_{ii}(q, \mu)$  also exhibit different behavior above and below the energy scale  $2\mu$ . When  $|\mathbf{q}| > 2\mu$ , we still use the instantaneous approximation and have

$$\Pi_{ii}(\mathbf{q}, \mu) = \frac{\alpha|\mathbf{q}|}{8}. \quad (23)$$

When  $|\mathbf{q}| < 2\mu$ ,  $\Pi_{ii}(q, \mu)$  vanishes at zero energy  $q_0 = 0$ , so it is not appropriate to use the instantaneous approximation. The energy dependence of  $\Pi_{ii}(q, \mu)$  should be explicitly maintained [22]. As pointed out in Ref. [22], the fermion self-energy is dominated by the low-energy region of  $q_0 \ll |\mathbf{q}| \ll 2\mu$ . We notice this approximation is widely used in the calculations of fermion self-energy due to gauge interaction [41, 43, 44, 47, 48] and critical ordering fluctuation [64]. In this region,  $\Pi_{ii}(q, \mu)$  can be approximated as

$$\Pi_{ii}(q_0, \mathbf{q}, \mu) = \frac{\alpha\mu q_0}{2\pi |\mathbf{q}|}. \quad (24)$$

At the lowest energy  $q_0 = 0$ , we have

$$\Pi_{ii}(q_0 = 0, \mathbf{q}, \mu) = 0. \quad (25)$$

This fact implies that, the transverse component of gauge interaction remains long-ranged even when the dynamical screening effect is taken into account. It also explains why the instantaneous approximation can not be used in this region. Physically, the long-range property of the transverse gauge interaction is protected by the local gauge invariance.

The fermion self-energy is given by

$$\begin{aligned} \Sigma_B &= \frac{\alpha}{N} \int_{\Lambda_1}^{\Lambda_0} \frac{d^3 q}{(2\pi)^3} \gamma_\mu \frac{(\not{k} - \not{q})}{(k - q)^2} \gamma_\nu \frac{B_{\mu\nu}}{q^2 + \Pi_B} \\ &= \frac{\alpha}{N} \int_{\Lambda_1}^{\Lambda_0} \frac{d^3 q}{(2\pi)^3} \frac{-\gamma_0(k_0 - q_0) + v_F \gamma \cdot (\mathbf{k} - \mathbf{q})}{(k - q)^2} \frac{1}{q^2 + \Pi_{ii}} \\ &\quad + \frac{\alpha}{N} \int_{\Lambda_1}^{\Lambda_0} \frac{d^3 q}{(2\pi)^3} \frac{-2v_F \gamma_1 k_1 q_1^2 - 2v_F \gamma_2 k_2 q_2^2}{\mathbf{q}^2(k - q)^2} \frac{1}{q^2 + \Pi_{ii}} \end{aligned} \quad (26)$$

$$\begin{aligned} &= (\gamma_0 k_0 \Sigma_{B0} + v_F \gamma_i k_i \Sigma_{B1})_{\text{fir}} \\ &\quad + (\gamma_0 k_0 \Sigma_{B0} + v_F \gamma_i k_i \Sigma_{B1})_{\text{sec}}. \end{aligned} \quad (27)$$

According to the general RG scheme [59], one needs to integrate out the high-energy degrees of freedom (fast modes) step by step, until eventually reaching the lowest energy. Since  $\Pi_{ii}$  behaves differently at high and low energies, the self-energy should be calculated separately.

For small external momentum  $k \ll \Lambda_1$ , we are allowed to make the approximation [60],  $(k - q)^2 \approx q^2$ , and write the first term of Eq. (27) as

$$\begin{aligned} (\Sigma_{B1} - \Sigma_{B0})_{\text{fir}} &\approx \frac{2\alpha}{N} \int_{\Lambda_1}^{\Lambda_0} \frac{d^3 q}{(2\pi)^3} \frac{1}{q^2} \frac{1}{(q^2 + \Pi_{ii})} \\ &\approx \frac{2\alpha}{N} \int_{\Lambda_1}^{\Lambda_0} \frac{d^3 q}{(2\pi)^3} \frac{1}{q^2} \frac{1}{q^2 + \frac{\alpha|\mathbf{q}|}{8}}. \end{aligned} \quad (28)$$

From RG theory, we know that the fermion velocity can receive singular renormalization only when the self-energy contains a logarithmic term. One can check that there is no such term in the regime where  $\frac{\alpha|\mathbf{q}|}{8} \ll q^2$ . Nevertheless, a logarithmic term emerges as one goes to the low-energy regime where  $\frac{\alpha|\mathbf{q}|}{8} \gg q^2$ . After neglecting  $q^2$ , it is straightforward to obtain

$$\begin{aligned} (\Sigma_{B1} - \Sigma_{B0})_{\text{fir}} &\approx \frac{2\alpha}{N} \int_{\Lambda_1}^{\Lambda_0} \frac{d^3 q}{(2\pi)^3} \frac{1}{q^2} \frac{1}{\frac{\alpha|\mathbf{q}|}{8}} \\ &= \frac{4}{\pi N} \ln \frac{\Lambda_0}{\Lambda_1}. \end{aligned} \quad (29)$$

Paralleling the above analysis, the second term of Eq. (26) can be computed in an analogous manner:

$$\begin{aligned} &-2 \frac{\alpha}{N} \int \frac{d^3 q}{(2\pi)^3} \frac{q_{1,2}^2}{\mathbf{q}^2(k - q)^2} \frac{1}{q^2 + \Pi_{ii}} \\ &\approx -2 \frac{\alpha}{N} \int \frac{d^3 q}{(2\pi)^3} \frac{q_{1,2}^2}{\mathbf{q}^2 q^2} \frac{1}{\Pi_{ii}} \\ &= -2 \frac{\alpha}{N} \int \frac{d^3 q}{(2\pi)^3} \frac{q_{1,2}^2}{\mathbf{q}^2 q^2} \frac{8}{\alpha|\mathbf{q}|} \\ &= -\frac{2}{\pi N} \ln \frac{\Lambda_0}{\Lambda_1}. \end{aligned} \quad (30)$$

Now the total self-energy is

$$\begin{aligned} \Sigma_{B1} - \Sigma_{B0} &= (\Sigma_{B1} - \Sigma_{B0})_{\text{fir}} + (\Sigma_{B1} - \Sigma_{B0})_{\text{sec}} \\ &= \frac{4}{\pi N} \ln \frac{\Lambda_0}{\Lambda_1} - \frac{2}{N\pi} \ln \frac{\Lambda_0}{\Lambda_1} \\ &= \frac{2}{N\pi} \ln \frac{\Lambda_0}{\Lambda_1}. \end{aligned} \quad (31)$$

This result is valid only when  $\Lambda_0 > \Lambda_1 > 2\mu$ . As the energy scale decreases below  $2\mu$ ,  $\Pi_{ii}$  should be replaced by its low-energy expression. In this case, the first term of fermion self-energy becomes

$$\begin{aligned} (\Sigma_{B1} - \Sigma_{B0})_{\text{fir}} &\approx \frac{2}{N} \int_{\Lambda_1}^{\Lambda_0} \frac{d^3 q}{(2\pi)^3} \frac{1}{q^2} \frac{1}{q^2 + \Pi_{ii}} \\ &\approx \frac{2\alpha}{N} \int_{\Lambda_1}^{\Lambda_0} \frac{d^3 q}{(2\pi)^3} \frac{1}{q^2} \frac{1}{q^2 + \frac{\mu q_0}{2\pi |\mathbf{q}|}}. \end{aligned} \quad (32)$$

In order to simplify calculations, we can divide momenta, frequency, and chemical potential by  $\alpha$  to make all these variables dimensionless. However, for notational convenience, we still denote  $\Lambda_{0,1}/\alpha$  as  $\Lambda_{0,1}$ . Now, the above equation is rewritten as

$$(\Sigma_{B1} - \Sigma_{B0})_{\text{fir}} = \frac{2}{N} \int_{\Lambda_1}^{\Lambda_0} \frac{d^3 q}{(2\pi)^3} \frac{1}{q^2} \frac{1}{q^2 + \frac{\mu q_0}{2\pi |\mathbf{q}|}}. \quad (33)$$

In the low-energy region  $q_0 \ll |\mathbf{q}| \ll 2\mu$ , this integral becomes

$$(\Sigma_{B1} - \Sigma_{B0})_{\text{fir}} \approx \frac{2}{N} \int dq_0 \int \frac{d|\mathbf{q}|}{(2\pi)^2} \frac{1}{|\mathbf{q}|^3 + \frac{\mu}{2\pi} q_0}$$

$$\begin{aligned}
& \times \theta(\Lambda_0^2 - q_0^2 - \mathbf{q}^2) \theta(q_0^2 + \mathbf{q}^2 - \Lambda_1^2) \\
& \approx \frac{2}{N} \int d q_0 \int \frac{d|\mathbf{q}|}{(2\pi)^2} \frac{1}{|\mathbf{q}|^3 + \frac{\mu}{2\pi} q_0} \\
& \times \theta(\Lambda_0^2 - \mathbf{q}^2) \theta(\mathbf{q}^2 - \Lambda_1^2) \\
& = \frac{2}{N} \int_0^{\Lambda_1} d q_0 \int_{\Lambda_1}^{\Lambda_0} \frac{d|\mathbf{q}|}{(2\pi)^2} \frac{1}{|\mathbf{q}|^3 + \frac{\mu}{2\pi} q_0}.
\end{aligned} \tag{34}$$

It seems easier to first integrate over  $q_0$  and then integrate over  $\mathbf{q}$ ,

$$\begin{aligned}
(\Sigma_{B1} - \Sigma_{B0})_{\text{fr}} & \approx \frac{4\pi}{N\mu} \int_{\Lambda_1}^{\Lambda_0} \frac{d|\mathbf{q}|}{(2\pi)^2} \ln \left( 1 + \frac{\frac{\mu}{2\pi} \Lambda_1}{|\mathbf{q}|^3} \right) \\
& \approx \frac{2}{8\pi^2 N} \ln \left( \frac{\Lambda_0}{\Lambda_1} \right) + \text{others terms}.
\end{aligned} \tag{35}$$

Analogously, we have

$$(\Sigma_{B1} - \Sigma_{B0})_{\text{sec}} = -\frac{1}{8\pi^2 N} \ln \left( \frac{\Lambda_0}{\Lambda_1} \right) + \text{others terms}. \tag{36}$$

We finally obtain the total contribution

$$\begin{aligned}
\Sigma_{B1} - \Sigma_{B0} & = (\Sigma_{B1} - \Sigma_{B0})_{\text{fr}} + (\Sigma_{B1} - \Sigma_{B0})_{\text{sec}} \\
& = \frac{1}{8\pi^2 N} \ln \left( \frac{\Lambda_0}{\Lambda_1} \right).
\end{aligned} \tag{37}$$

Here we keep only the logarithmic term, which survives at the lowest energy and corresponds to the stable fixed point produced by the long-ranged transverse gauge interaction.

Summarizing the above results, we obtain the following RG equation

$$k \frac{dv_F(k)}{dk} = \gamma_v v_F(k), \tag{38}$$

where

$$\gamma_v = \frac{1}{8\pi^2 N}. \tag{39}$$

This equation has the following solution,

$$v_F \propto k^{\gamma_v}, \tag{40}$$

where the constant  $\gamma_v$  defines an anomalous dimension of velocity  $v_F$  and the dimensionless momenta  $k$  actually corresponds to  $k/\Lambda$  with  $\Lambda$  being the UV cutoff. Obviously, the constant fermion velocity becomes strongly momentum dependent due to the long-range transverse gauge interaction. The  $k$ -dependence of fermion velocity is shown in Fig. (1) at both zero and finite  $\mu$ . If we define  $v_F(\Lambda_0)$  as the bare velocity, which is taken to be unity in the figure, then the effective velocity  $v_F(k)$  decreases monotonically when  $k$  is lowering. It eventually vanishes as  $k \rightarrow 0$ .

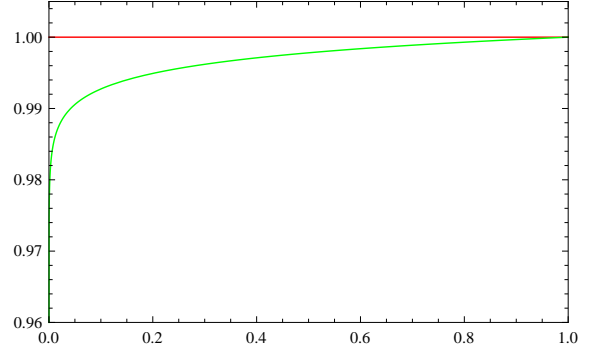


FIG. 1: Momenta  $k$  is rescaled by ultraviolet cutoff,  $k \rightarrow k/\Lambda$ . The upper curve represents the constant velocity at  $\mu = 0$ . The lower curve shows the  $k$ -dependence of renormalized velocity at  $\mu \neq 0$ .

After this renormalization, the energy spectrum of Dirac fermions becomes

$$\epsilon(k) \propto k^{1+\gamma_v}. \tag{41}$$

This type of unusual velocity renormalization is a characteristic feature of Dirac fermion systems, and has been studied extensively in the contexts of  $d$ -wave high-temperature superconductor [27, 61, 62] and graphene [60, 65, 66]. The finite anomalous dimension of fermion velocity distinguishes the system at finite  $\mu$  from that at zero  $\mu$ . An interesting issue is how this anomalous dimension affects the observable quantities of Dirac fermions, which will be addressed in the following two sections.

It is interesting to note that the anomalous dimension  $\gamma_v$  is a universal constant for any given flavor  $N$ . Although  $\gamma_v$  is finite only at finite  $\mu$ , it does not explicitly depend on  $\mu$ . It is the finiteness, rather than the precise value, of  $\mu$  that is really important. A finite  $\gamma_v$  is generated as long as  $\mu$  becomes finite, no matter how small it is. Therefore, the ground state changes fundamentally once one moves away from the neutral Dirac point.

### III. SPECIFIC HEAT OF DIRAC FERMIONS

According to Landau FL theory, a weakly interacting fermion system is in one-to-one correspondence with a non-interacting fermion gas. There is a sharp Fermi surface in a FL with well-defined quasiparticles existing in the low energy regime. The properties of a FL can be manifested in a variety of physical quantities, such as spectral function, specific heat, DOS, and susceptibility. The Coulomb interaction does not destroy the stability of FL state in normal metals, because it is always statically screened by the collective particle-hole excitations. However, non-FL behavior would emerge when there is some kind of long-range gauge interaction. We now would like to examine the corrections of gauge interaction to several physical quantities of massless Dirac fermions. We will consider specific heat in this section, and consider DOS and compressibility in the next section.

In order to calculate specific heat, we first need to calculate the free energy. In the following, to facilitate calculations we make the rescaling transformations:  $T, k, \mu \rightarrow T/\Lambda, k/\Lambda, \mu/\Lambda$ , where  $\Lambda$  is ultraviolet cutoff. When  $\mu = 0$ , the specific heat due to gauge interaction has already been studied in previous works [37, 38], which show that  $C_V \propto T^2$ .

The free energy density will be computed using the methods given in Ref. [67]. The partition function is

$$Z = \prod_{n, \mathbf{k}, \alpha} \int \mathcal{D}[-i\psi_{\alpha, n}^\dagger(\mathbf{k})] \mathcal{D}[\psi_{\alpha, n}(\mathbf{k})] e^S, \quad (42)$$

where the action is expressed as

$$\begin{aligned} S &= \sum_{n, \mathbf{k}} [-i\psi_{\alpha, n}^\dagger(\mathbf{k})] D_{\alpha, \rho} [\psi_{\rho, n}(\mathbf{k})], \\ D &= \beta [-(\omega_n - i\mu) - iv_F \gamma^0 \gamma \cdot \mathbf{k}]. \end{aligned} \quad (43)$$

Employing the functional integral formula for Grassmann variables,

$$\int \mathcal{D}[\eta^\dagger] \mathcal{D}[\eta] e^{\eta^\dagger D \eta} = \det D, \quad (44)$$

it is easy to obtain

$$\begin{aligned} \ln Z &= \sum_{n, \mathbf{k}} \ln \det D(n, \mathbf{k}) \\ &= \sum_{n, \mathbf{k}} \ln \left\{ \beta^4 [(\omega_n - i\mu)^2 + v_F^2 (k_1^2 + k_2^2)]^2 \right\} \\ &= \sum_{n, \mathbf{k}} \ln \left\{ \beta^4 [(\omega_n - i\mu)^2 + \epsilon(k)^2]^2 \right\} \\ &= \sum_{n, \mathbf{k}} \left\{ \ln [\beta^2 (\omega_n^2 + (\epsilon(k) - \mu)^2)] \right. \\ &\quad \left. + \ln [\beta^2 (\omega_n^2 + (\epsilon(k) + \mu)^2)] \right\}. \end{aligned} \quad (45)$$

Applying the identities,

$$\int_1^{\beta^2(\omega \pm \mu)^2} \frac{d\theta^2}{\theta^2 + (2n+1)^2 \pi^2} + \ln[1 + (2n+1)^2 \pi^2] = \ln[(2n+1)^2 \pi^2 + \beta^2(\omega \pm \mu)^2], \quad (46)$$

$$\sum_{n=-\infty}^{\infty} \frac{1}{(n-x)(n-y)} = \frac{\pi [\cot(\pi x) - \cot(\pi y)]}{y-x}, \quad (47)$$

we are left with

$$\begin{aligned} \ln Z &= 2V \int \frac{d^2 \mathbf{k}}{(2\pi)^2} \left[ \beta \epsilon(k) + \ln \left( 1 + e^{-\beta(\epsilon(k) - \mu)} \right) \right. \\ &\quad \left. + \ln \left( 1 + e^{-\beta(\epsilon(k) + \mu)} \right) \right] \end{aligned} \quad (48)$$

after some straightforward algebra. Since the free energy density  $f = \frac{F}{V} = -\frac{1}{\beta V} \ln Z$ , we have

$$f(T, \gamma_v, \mu) = -2T \int \frac{d^2 \mathbf{k}}{4\pi^2} \ln \left[ 1 + \exp \left( -\frac{\epsilon(k) \pm \mu}{T} \right) \right], \quad (49)$$

where the zero-point energy (the first term in  $\ln Z$ ) has been discarded. The fermion specific heat can be obtained from free energy, namely

$$C_V = -T \frac{\partial^2 f}{\partial T^2}. \quad (50)$$

In the absence of gauge interaction, the fermion velocity takes its bare value and the anomalous dimension  $\gamma_v = 0$ . Therefore, the free energy density is simply

$$f^0(\mu) = -2T \int \frac{d^2 \mathbf{k}}{4\pi^2} \left[ \ln \left( 1 + e^{-\frac{k+\mu}{T}} \right) + \ln \left( 1 + e^{-\frac{k-\mu}{T}} \right) \right]. \quad (51)$$

It is easy to integrate over  $\mathbf{k}$ , and get

$$f^0(\mu) = \frac{T^3}{\pi} \left[ \text{Li}_3 \left( -e^{\frac{\mu}{T}} \right) + \text{Li}_3 \left( -e^{-\frac{\mu}{T}} \right) \right], \quad (52)$$

where  $\text{Li}_3(z)$  is polylogarithmic function. The specific heat has the following form,

$$\begin{aligned} C_V^0(\mu) &= \frac{1}{\pi} \left\{ \mu^2 \ln \left[ \left( 1 + e^{-\frac{\mu}{T}} \right) \left( 1 + e^{\frac{\mu}{T}} \right) \right] \right. \\ &\quad \left. - 6T^2 \left[ \text{Li}_3 \left( -e^{\frac{\mu}{T}} \right) + \text{Li}_3 \left( -e^{-\frac{\mu}{T}} \right) \right] \right. \\ &\quad \left. - 4\mu T \left[ \text{Li}_2 \left( -e^{-\frac{\mu}{T}} \right) - \text{Li}_2 \left( -e^{\frac{\mu}{T}} \right) \right] \right\}. \end{aligned} \quad (53)$$

It is obvious that the specific heat  $C_V^0 = \frac{9\zeta(3)}{\pi} T^2 \propto T^2$  at  $\mu = 0$ . At finite  $\mu$ , the transverse gauge interaction induces an anomalous dimension for fermion velocity, which modifies the Dirac fermion energy spectrum. Now the free energy density becomes

$$f(\mu, \gamma_v) = -2T \int \frac{k dk}{2\pi} \ln \left[ \left( 1 + e^{-\frac{k+\mu}{T}} \right) \left( 1 + e^{-\frac{k-\mu}{T}} \right) \right], \quad (54)$$

where  $\eta = 1 + \gamma_v$ . It is convenient to define  $x = k^\eta$ , and write the free energy as

$$f(\mu, \eta) = -\frac{T}{\pi \eta} \int dx x^{\frac{2-\eta}{\eta}} \ln \left[ \left( 1 + e^{-\frac{x+\mu}{T}} \right) \left( 1 + e^{-\frac{x-\mu}{T}} \right) \right]. \quad (55)$$

Making derivatives of  $f(\mu, \eta)$  with respect to  $T$ , we obtain

$$C_V = \frac{1}{\pi \eta T^2} \int dx x^{\frac{2-\eta}{\eta}} \left[ \frac{(x+\mu)^2 e^{\frac{x+\mu}{T}}}{\left( 1 + e^{\frac{x+\mu}{T}} \right)^2} + \frac{(x-\mu)^2 e^{\frac{x-\mu}{T}}}{\left( 1 + e^{\frac{x-\mu}{T}} \right)^2} \right], \quad (56)$$

which is complicated and will be evaluated numerically.

Note the UV cutoff  $\Lambda$  does not qualitatively affect our basic conclusion, which allows us to set  $\Lambda = 1$ . The specific heat  $C_V$  explicitly depends on both chemical potential  $\mu$  and temperature  $T$ . Its  $T$ -dependence shown in Fig. (2) for  $N = 4$ . When chemical potential  $\mu = 0$ ,

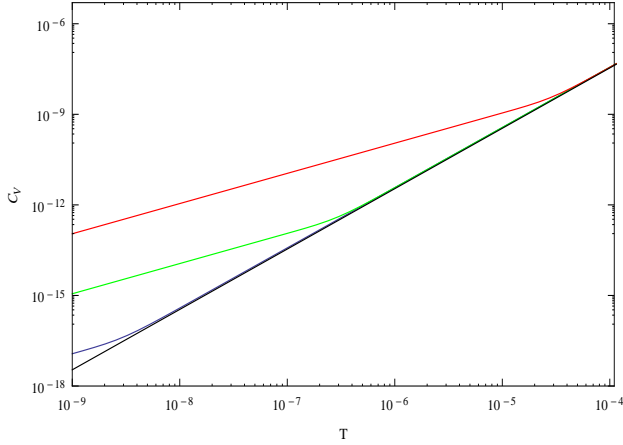


FIG. 2:  $\gamma_v = \frac{1}{8\pi^2 N}$ . The curves from top to bottom are  $C_V(\mu = 10^{-4})$ ,  $C_V(\mu = 10^{-6})$ ,  $C_V(\mu = 10^{-8})$ , and  $C_V^0 = \frac{9\zeta(3)}{\pi}T^2$ , respectively.

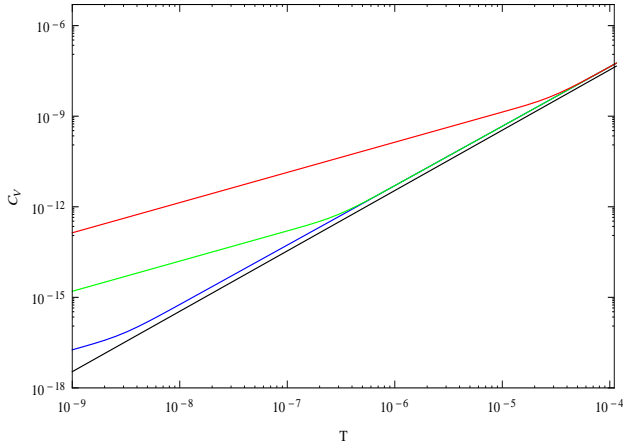


FIG. 3:  $\gamma_v \rightarrow 5\gamma_v$ . The curves from top to bottom are  $C_V(\mu = 10^{-4})$ ,  $C_V(\mu = 10^{-6})$ ,  $C_V(\mu = 10^{-8})$ , and  $C_V^0 = \frac{9\zeta(3)}{\pi}T^2$ , respectively.

the specific heat behaves as  $C_V \propto T^2$ . At finite  $\mu$ , the specific heat deviates from the  $T^2$  curve. The deviation becomes more significant for larger  $\mu$ . At low temperature, the specific heat can be approximately written as power-law,  $C_V \propto T^\delta$ , where the exponent  $\delta$  is a function of  $\mu$ . Such unconventional non-FL behavior arises from the anomalous dimension of fermion velocity,  $\gamma_v$ , which is generated by the long-ranged transverse gauge interaction at finite  $\mu$ . In particular, the deviation occurs once  $\mu$  becomes finite. This implies that the ground states of QED<sub>3</sub> are very different at zero and finite  $\mu$ .

In many works on non-FL behavior caused by singular interactions, the specific heat is expressed in a logarithmic function [27, 41, 42, 46, 65, 66], i.e.,  $C_V \propto T \ln T$ . Note that this logarithmic expression does not contradict our results. Actually, the power-law specific heat presented here amounts to a summation of all powers of  $\ln T$  [56, 57, 60, 68].

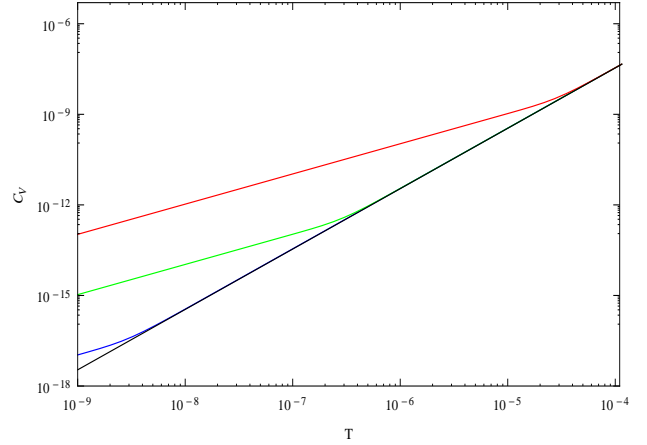


FIG. 4:  $\gamma_v \rightarrow \frac{1}{5}\gamma_v$ . The curves from top to bottom are  $C_V(\mu = 10^{-4})$ ,  $C_V(\mu = 10^{-6})$ ,  $C_V(\mu = 10^{-8})$ , and  $C_V^0 = \frac{9\zeta(3)}{\pi}T^2$ , respectively.

In the above calculations, the anomalous dimension  $\gamma_v$  plays an essential role. However, it is obtained by adopting certain approximations. To examine the reliability of our results, we suppose  $\gamma_v = \frac{1}{8\pi^2 N}$  becomes  $5\gamma_v$  or  $\gamma_v/5$  after including higher order corrections, and show the corresponding results in Fig. (3) and Fig. (4). Apparently, the basic conclusions are independent of the precise value of anomalous dimension.

#### IV. DENSITY OF STATES AND COMPRESSIBILITY

We now turn to the interaction corrections to DOS and compressibility. At  $\mu = 0$ , the propagator of massless Dirac fermion is

$$G(i\omega, \mathbf{k}) = \frac{1}{i\omega\gamma_0 - v_F\gamma \cdot \mathbf{k}} = \frac{i\omega\gamma_0 + v_F\gamma \cdot \mathbf{k}}{(i\omega)^2 - v_F^2\mathbf{k}^2}. \quad (57)$$

After analytical continuation,  $i\omega \rightarrow \omega + i\delta$ , we have the following retarded propagator

$$G^{\text{ret}}(\omega, \mathbf{k}) = \frac{\omega\gamma_0 + v_F\gamma \cdot \mathbf{k}}{\omega^2 - v_F^2\mathbf{k}^2 + i\text{sgn}(\omega)\delta}. \quad (58)$$

The corresponding spectral function is given by

$$\begin{aligned} A(\omega, \mathbf{k}) &= -\frac{1}{\pi} \text{Im} G^{\text{ret}}(\omega, \mathbf{k}) \\ &= \text{sgn}(\omega)(\omega\gamma_0 - v_F\gamma \cdot \mathbf{k})\delta(\omega^2 - v_F^2\mathbf{k}^2), \end{aligned} \quad (59)$$

which then gives rise to the DOS,

$$\begin{aligned} \rho(\omega) &= N \int \frac{d^2\mathbf{k}}{(2\pi)^2} \text{Tr}\{\gamma_0 \text{Im} G^{\text{ret}}(\omega, \mathbf{k})\} \\ &= \frac{N\omega}{v_F^2\pi}. \end{aligned} \quad (60)$$

Apparently, the DOS vanishes at the Fermi level,  $\omega = 0$ .

At finite chemical potential  $\mu$ , the fermion propagator becomes

$$G(i\omega, \mathbf{k}) = \frac{(i\omega + \mu)\gamma_0 + v_F \boldsymbol{\gamma} \cdot \mathbf{k}}{(i\omega + \mu)^2 - v_F^2 \mathbf{k}^2}. \quad (61)$$

The DOS can be calculated similarly, with the expression

$$\begin{aligned} \rho(\omega) &= N \int \frac{d^2 \mathbf{k}}{(2\pi)^2} \text{Tr}\{\gamma_0 \text{Im} G^{\text{ret}}(\omega, \mathbf{k})\} \\ &= \frac{N(\omega + \mu)}{v_F^2 \pi}, \end{aligned} \quad (62)$$

which approaches a constant proportional to  $\mu$  as  $\omega \rightarrow 0$ . We are interested in the gauge interaction corrections to the above expressions. At finite  $\mu$ , the fermion velocity  $v_F$  becomes  $k$ -dependent,  $v_F' \propto k^{\gamma_v}$ . Using this expression, we find that,

$$\begin{aligned} \rho(\omega) &= 4N\pi(\omega + \mu) \int \frac{dk^2}{(2\pi)^2} \delta((\omega + \mu)^2 - v_F'^2 k^2) \\ &= \frac{4N\pi(\omega + \mu)}{\eta v_F'^2} \int \frac{dk'}{(2\pi)^2} k'^{\frac{2-\eta}{\eta}} \delta((\omega + \mu)^2 - k'^2) \\ &= \frac{N(\omega + \mu)^{\frac{2-\eta}{\eta}}}{\eta v_F'^2 \pi}. \end{aligned} \quad (63)$$

The effect of gauge interaction is reflected in the nontrivial exponent of DOS.

Since the difference between zero and finite chemical potential is of primary interest in our work, we only consider zero-temperature compressibility, which is an important quantity describing the electronic properties of any interacting system. In its original meaning, the compressibility is defined as  $\kappa = \partial V / \partial F$  [69], with  $V$  and  $F$  being the volume and compressing force, respectively. However, in practical many-particle calculations, it is more convenient to define the compressibility as  $\kappa = \partial n / \partial \mu$ , where  $n$  is the number of particles per area [66, 70]. The compressibility of massless Dirac fermions vanishes at zero chemical potential,  $\mu = 0$ . This behavior will be changed at finite density, i.e.,  $\mu \neq 0$  [66, 70]. In order to obtain  $\kappa$ , we only need to calculate the  $\mu$ -dependence of particle number  $n$ .

Using the DOS expressed in Eq. (62), the particle number in the absence of gauge interaction is

$$n = \int_{-\mu}^0 d\omega \rho(\omega) = \int_{-\mu}^0 \frac{N(\omega + \mu)}{v_F^2 \pi} d\omega = \frac{N\mu^2}{2v_F^2 \pi}, \quad (64)$$

which leads to

$$\kappa = \frac{\partial n}{\partial \mu} = \frac{N\mu}{v_F^2 \pi}. \quad (65)$$

After including the gauge interaction, the DOS is given by Eq. (63). Now the particle number becomes

$$n = \int_{-\mu}^0 d\omega \rho(\omega) = \int_{-\mu}^0 \frac{N(\omega + \mu)^{\frac{2-\eta}{\eta}}}{\eta v_F'^2 \pi} d\omega = \frac{N\mu^{\frac{2}{\eta}}}{2v_F'^2 \pi}, \quad (66)$$

which then yields

$$\kappa = \frac{N\mu^{\frac{2}{\eta}-1}}{\eta v_F'^2 \pi}. \quad (67)$$

Once again, the effect of gauge interaction is reflected in the exponent.

## V. SUMMARY

In summary, we have studied the effects of a finite chemical potential on the behavior of Dirac fermions in QED<sub>3</sub>. At zero chemical potential, there is no fermion velocity renormalization. At finite chemical potential, the longitudinal gauge interaction becomes short-ranged, but the transverse gauge interaction remains long-ranged and leads to singular velocity renormalization. An explicit calculation shows that a finite anomalous dimension of velocity is generated and gives rise to unconventional properties in some physical quantities, including specific heat, DOS, and compressibility. Therefore, the massless Dirac fermions behave quite differently at finite and zero chemical potential. This difference, together with the difference in fermion damping rate [21, 22], indicates that the ground state of QED<sub>3</sub> is fundamentally changed once the chemical potential becomes finite.

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